Analysis of Preventive Maintenance in Transactions Based Software Systems

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Abstract—Preventive maintenance of operational software systems, a novel technique for software fault tolerance, is used specifically to counteract the phenomenon of software “aging.” However, it incurs some overhead. The necessity to do preventive maintenance, not only in general purpose software systems of mass use, but also in safety-critical and highly available systems, clearly indicates the need to follow an analysis based approach to determine the optimal times to perform preventive maintenance.

In this paper, we present an analytical model of a software system which serves transactions. Due to aging, not only the service rate of the software decreases with time, but also the software itself experiences crash/hang failures which result in its unavailability. Two policies for preventive maintenance are modeled and expressions for resulting steady state availability, probability that an arriving transaction is lost and an upper bound on the expected response time of a transition are derived. Numerical examples are presented to illustrate the applicability of the models.

Index Terms—Preventive maintenance, software fault tolerance, software rejuvenation, transactions based software systems, reliability modeling, Markov regenerative models.

1 INTRODUCTION

It is now well established that system outages are caused more due to software faults than due to hardware faults [24], [11]. Given the current growth in software complexity and reuse, the trend is likely to grow. It is also well-known that, regardless of development, testing, and debugging time, software still contains some residual faults. Thus, fault tolerant software has become an effective alternative to virtually impossible fault-free software. The scope of this paper lies in the quantitative evaluation of a novel technique for software fault tolerance, viz., preventive maintenance of operational software systems. Although the use of preventive maintenance is common in physical systems, its potential effectiveness in enhancing software dependability has only recently been recognized. As we shall exemplify later, in certain situations, it is simply necessary.

Traditional methods of software fault-tolerance, namely N-version programming [2], recovery blocks [23], and N-self checking programming [19], are all based on design diversity. Primarily, the following two factors motivate the need to explore alternate techniques for software fault tolerance.

1) Reliability/Availability Versus Cost

The need for high reliability and availability is not just restricted to safety-critical systems [11]. Telephone switches [7], airline reservation systems, process and production control, stock trading systems, computerized banking, etc., all demand very high availability. A survey showed that computer downtime in non-safety critical systems cost over 3.8 Billion dollars in 1991 in the U.S. [25]. With the explosive increase in the popularity of network centric computing, web servers, too, need to be highly available. In most of these systems, the cost of providing fault tolerance via the use of multiple variants is prohibitive. With commercial considerations driving technology more than ever, release times of software are required to be less and less, forcing organizations to reduce testing and debugging cycle times. Further, with software reuse gaining popularity, many times it is simply not feasible to test the middleware and operating system on which the final software product is based. This leaves no option but to tolerate the residual faults during its operational phase.

2) Nature of software failures

More recently, from the study of field failure data, it has been observed that a large percentage of operational software failures are transient in nature [12], [13], [17], caused by phenomena such as overloads or timing and exception errors [24], [5]. A common characteristic of these type of failures is that, upon re-execution of the software, the failure does not recur. The error condition, which results in the failure, typically manifests itself in the operating environment of the executing software. Due to the complexity of modern-day operating systems and intermediate layer software, it has been observed that the same error condition, when the software is re-executed, is unlikely to recur, thus avoiding the failure.
A study done by Adams implies that the best approach to masking software faults is to simply restart the system [1]. Environment diversity, a generalization of restart, has been proposed as a cheap, yet effective, technique for software fault-tolerance [15], [18]. Typical transient failures occur because of design faults in software which result in unacceptable erroneous states in the OS environment of the process. The OS environment refers to resources that the program must access through the operating system, such as swap space, file systems, communication channels, keyboard, monitors, time, etc. [27]. The key idea behind environment diversity is to modify the operating environment of the running process. Typically, this has been done on a corrective basis, i.e., upon a failure, the software is restarted after some cleanup, which, in most cases, results in a different, error free OS environment state, thus avoiding further failure.

Recently, the phenomenon of software “aging” [16] has come to light, where such error conditions actually accrue with time and/or load. This observation has led to proposals of a pro-active approach to environment diversity in which the operational software is occasionally stopped and “cleaned up” to remove any potential error conditions. Since the preventive actions can be performed at suitable times (such as when there is no load on the system), it typically results in less downtime and cost than the corrective approach. Even so, it incurs some overhead and, if done more often than necessary, will result in higher downtime/cost. Therefore, an important research issue is to determine the optimal times to perform preventive maintenance of operational software systems.

In this paper, we present a stochastic model for a transactions based software system which employs preventive maintenance (henceforth referred to as PM). Three measures, the availability of the software to provide service, the probability of loss of a transaction, and the response time of a transaction, are considered. The model is developed under very general conditions and requires numerical solution. We compute each of the three measures under two policies for PM, which were proposed in [9], in the same framework. The rest of the paper is organized as follows. In Section 2, we present real life examples of aging and PM in software systems to illustrate the different forms in which they occur and to motivate the need for analysis of such systems. In Section 3, we describe the system model, along with the assumptions on modeling aging, failure, and PM policies. Section 4 is comprised of the analytical solution of the model for availability, loss probability, and response time measures. In Section 5, we illustrate the usefulness of the models via numerical examples. The two PM policies are compared along with the effect of model parameters on the derived measures. It is shown that the PM interval which maximizes availability may be very different from the PM interval which minimizes the probability of loss or the response time, indicating caution in the selection of the optimum PM interval. Finally, in Section 6, we present the conclusions.

2 Preventive Maintenance of Operational Software

While monitoring real applications, the phenomenon of software “aging” has been observed to result in performance degradation and/or transient failures.1 Failures of both crash/hang type, as well as those resulting in data inconsistency because of aging, have been reported. Memory bloating and leaking, unreleased file-locks, data corruption, storage space fragmentation, and accumulation of roundoff errors are some typical causes of slow degradation.

The widely used web browser “Netscape” is known to suffer from memory leaks which lead to occasional crash or hang of the application(s), especially in a computer with relatively low swap space. A similar memory leaking problem has been reported in the news-reader program “xrn.” All PC users are familiar with the occasional “switch off and on” of the computer to recover from hangs. Such examples of aging in software of mass use are probably just an inconvenience, but, in systems with high reliability/availability requirements, software aging can result in high cost. Huang et al. report this phenomenon in telecommunications billing applications where, over time, the application experiences a crash or a hang failure [16]. Avritzer and Weyuker have witnessed aging in transaction processing software systems where the effect manifests as gradual performance degradation [3]. The service rate of the software decreases with time, increasing queue lengths and, eventually, starts losing packets. Perhaps the most vivid example of aging can be found in [21], where the failure resulted in loss of human life. Patriot missiles, used during the Gulf war to destroy Iraq’s Scud missiles, used a computer whose software accumulated error. The effect of aging in this case was misinterpretation of an incoming Scud as not a missile but just a false alarm, which resulted in the death of 28 U.S. soldiers.

Huang et al. [16] have proposed the technique of Software Rejuvenation, which simply involves stopping the running software occasionally, removing the accrued error conditions, and restarting the software. Garbage collection, flushing operating system kernel tables, and reinitializing internal data structures are some examples of what cleaning the internal state of a software might involve. An extreme, but well-known, example of rejuvenation is a hardware reboot. It has been implemented in the real-time system collecting billing data for most telephone exchanges in the U.S. [4]. A very similar technique has been used by Avritzer and Weyuker in a large transaction processing software system [3], where the server is rebooted occasionally, upon which, its service rate is restored to the peak value. They call it software capacity restoration. Both of the above independently proposed techniques are specific cases of PM performed on operational software. Grey [14] proposed performing operations solely for fault management in SDI (Strategic Defense Initiative) software, which are invoked

1. It should be noted that the term “software aging” was used in [1] to mean degradation in the quality of the software by an increase in number or severity of design faults due to repeated bug fixes, which is different from our meaning. Perhaps, in our context, “process aging” is more appropriate, but we choose to keep the term software aging to be consistent with the usage by Huang et al. [16].
whether or not the fault exists, and called it “operational redundancy.” Tai et al. [26] have proposed and analyzed the use of on-board PM for maximizing the probability of successful mission completion of spacecraft with very long mission times. In a safety critical environment also, the necessity of performing PM is evident from the example of aging in Patriot’s software [21]. In the words of the author, “On 21 February, the office sent out a warning that ‘very long running time’ could affect the targeting accuracy. The troops were not told, however, how many hours ‘very long’ was, or that it would help to switch the computer off and on again after 8 hours.”

In some cases, PM is performed on a per process basis, while, in others, a system-wide maintenance is done. In each case, however, the maintenance incurs an overhead which should be balanced against the cost incurred due to unexpected outage caused because of a failure. This, in turn, demands a quantitative analysis, which, in the context of software systems, has only recently started getting attention. Bernstein stresses the need for emergence of a new field, “software dynamics” [4], and calls for “developing the design constraints, analytically, to make software behavior periodic and stable in its operational phase.”

The contribution of this paper lies in presenting an analytical model for transaction based software which experiences aging and employs PM to avoid unexpected outages. Furthermore, two policies for PM are analyzed:

1) Purely time-based: PM is performed at fixed deterministic interval, and

2) Time and load-based: PM is attempted at fixed intervals and performed only if the software is currently not serving any transactions.

In both cases, equations for the steady state availability, the long run probability of a transaction loss, and an upper bound on the mean response time are derived.

2.1 Previous Work

The single most important factor (as will be shown via numerical examples) in determining the accuracy of such a model is the assumptions made in capturing aging. Primarily, assumptions regarding the following aspects of aging need to be made:

- **Effect of Aging:** Effect of aging has been witnessed as crash/hang failure, which results in unavailability of the software, and gradual performance degradation.²

User perceivable impact of one may be more dominant than the other, but, typically, both are present to some degree in a software which experiences aging. In [16], [8], [9], [10], [26], only the failures causing unavailability of the software are considered, while, in [22], only a gradually decreasing service rate of a software which serves transactions is assumed. In this paper, we consider both the effects together in a single model.

- **Distribution:** There is no consensus on the time to failure distribution of an operational software and of the nature of service degradation it experiences. Therefore, for wide applicability, it is essential that a model be able to accommodate general distributions and not be restricted to predetermined ones. This way, with the availability of data, a specific distribution can then be applied on a per system basis. Models proposed in [16], [8], [9] are restricted to hypo-exponentially distributed time to failure. Those proposed in [10], [22], [26] can accommodate general distributions, but only for the time to failure. In our model, we allow for generally distributed time to failures, as well as for the service rate to be an arbitrary function of time.

- **Dependence on Load:** None of the previous studies capture the effect of load on aging. As it has been noted that transient failures are partly caused by overload conditions [24], in our model, we allow for the failure and the service rates to be functions of time, instantaneous load, mean accumulated load, or a combination of the above.

Table 1 summarizes the differences in capturing the effect of aging and on the assumptions in the distribution and dependence of these effects in previous work. It also shows the differences in the measures evaluated. In [10] and [26], software with a finite mission time is considered. Mean completion time in the presence of aging failures is computed in [10], whereas, in [26], the probability of successful completion by the mission deadline is computed. In the rest, [16], [8], [9], as well as in this paper, measures of interest in a transaction based software intended to run forever are evaluated. Where, in [16] and [8], only the steady state availability is computed, both steady state availability, as well as the long run probability that a transaction is denied service, are computed in [9]. In this paper, we evaluate the steady state availability, the probability of loss of a transaction, as well as an upper bound on the mean response time of a transaction. Optimizing one may result in an unacceptable value for the other. Optimal selection based on

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<th>Model load dependence?</th>
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Our | X   | X   | X   | X   |

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² Incorrect output because of data inconsistency, from a modeling standpoint, can be captured in the model for crash/hang failure, since it is simply a failure at a specific time point.
constraints on one or more of the measures can then be made via solution of our model. Last, all previous models except [10] and [26] are just special cases of the model presented in this paper.

The rest of the paper deals with the proposed model, evaluation of the three measures, and numerical examples.

3 SYSTEM MODEL

The system we study consists of a server type software to which transactions arrive at a constant rate \(\lambda\). Each transaction receives service for a random period. The service rate of the software is an arbitrary function measured from the last renewal of the software (because of aging), denoted by \(\mu(t)\). Therefore, a transaction which starts service at time \(t\), occupies the server for a time whose distribution is given by

\[
1 - e^{-\int_0^t \mu(u)\, du}
\]

\(\mu(t)\) can be a constant, a function of time \(t\), a function of the instantaneous load on the system, a function of total processing done in a given interval, or a combination of the above. We shall defer the explicit specification of the parameter in \(\mu(t)\) until Section 3.1.

If the software is busy processing a transaction, arriving customers are queued. Total number of transactions that the software can accommodate is \(K\) (including the one being processed) and any more arriving when the queue is full are lost. The service discipline is FCFS. This state, in which the software is available for service (albeit with decreasing service rate), is denoted as state “A” (see Fig. 1).

Further, the software can fail, upon which, recovery procedure is started. This state, in which the software is recovering and is unavailable for service, is denoted as state “B.” The rate at which it fails, i.e., at which the software moves from state \(A\) to state \(B\), is denoted by \(\rho(t)\). Let the time to failure be denoted by random variable \(X\). Then, its distribution is given by

\[
F_X(t) = 1 - e^{-\int_0^t \rho(u)\, du}
\]

Like \(\mu(t)\), \(\rho(t)\) can also be function of time, instantaneous load, mean accumulated load, or a combination of the above. Explicit specification of \(\rho(t)\) is deferred until Section 3.1.

The effect of aging, therefore, may be captured by using decreasing service rate and increasing failure rate, where the decrease or the increase, respectively, can be a function of time, instantaneous load, mean accumulated load, or a combination of the above. The service degradation and hang/crash failures in our model are assumed to be stochastically independent processes. Their interdependence, if it exists in the real system, can be modeled by using parametric dependence in the definitions of \(\rho(t)\) and \(\mu(t)\). The failure process is stochastically independent of the arrival process and any transactions in the queue at the time of failure are assumed to be lost. Moreover, any transactions which arrive while recovery is in progress are also lost. We note that our assumptions regarding the loss of transactions may not hold true for database systems with logging capabilities. However, they are true for a large class of software systems providing real-time services. The most notable example is a web server. Requests arriving to such software systems are typically assumed to be of equal criticality. Time to recover from a failed state is denoted by \(Y_f\) with associated general distribution \(F_{Y_f}\).

Last, the software occasionally undergoes PM. This state is denoted as state “C.” PM is allowed only from state “A.” We consider two different policies which determine the time to perform PM.

1) Purely time based. Under this policy, henceforth referred to as Policy I, PM is initiated after a constant time \(\delta\) has elapsed since it was started (or restarted). We shall refer to \(\delta\) under this policy as the PM interval.

2) Instantaneous load and time based. Under this policy, henceforth referred to as Policy II, a constant waiting period \(\delta\) must elapse before PM is attempted. Further, after this time, PM is initiated if and only if there are no transactions in the system. \(\delta\) under this policy shall be referred to as PM wait. The actual PM interval under Policy II is determined by the sum of PM wait and the time it takes for the queue to get empty from that point onward. The latter quantity is dependent on system parameters and cannot be controlled. The actual PM interval, therefore, has a range \([\delta, \infty)\).

Regardless of the policy used, it takes a random amount of time, denoted by \(Y_r\), to perform PM. Let \(F_{Y_r}\) be its distribution. As will be shown in the following section, our model does not require any assumptions on the nature of \(F_{Y_f}\) and \(F_{Y_r}\). Only the respective expectations, \(E[Y_f]\) and \(E[Y_r]\) are assumed to be finite.

Once recovery from the failed state or PM is complete, the software is reset in state \(A\) and is as good as new. From this moment, which constitutes a renewal, the whole process stochastically repeats itself. The transition behavior of the software among states \(A, B, C\) and \(C\) is illustrated in Fig. 1.

The queuing behavior of the software, on the other hand, as determined by the two PM policies, is illustrated in Fig. 2. The horizontal axis represents time \(t\) and the vertical axis represents the number of transactions queued in the software at time \(t\), denoted by \(N(t)\). Fig. 2a shows a sample path in which PM is initiated as soon constant time \(\delta\) elapses. In accordance with Policy I, the transactions already in the queue at time \(\delta\) are lost.

Fig. 2b illustrates Policy II where, at time \(\delta\), some transactions are in the queue, i.e., \(N(\delta) > 0\). In this case, the software waits until the queue is empty, upon which, PM is initiated. This wait is a random quantity, denoted in the

3. A generalization of Policy II in which, after time \(\delta\), PM is initiated if and only if the number of transactions in the queue goes below a certain threshold can easily be accommodated in our model.

Fig. 1. Macro-states representation of the software behavior.
figure by $B$. Intuitively, if $B$ is very large, it is likely that the software will fail before it has a chance to undergo PM.

### 3.1 How to Capture Aging?

The effects of aging, i.e., degradation in performance and failures causing unavailability, are present in varying severity in different systems. Further, each of them may be influenced by different operating parameters in different software systems. We now show how, in our model, the varying severity and dependencies may be captured via proper choice of parameters of $\mu(\cdot)$ and $\rho(\cdot)$. Flexibility in this choice in the same framework widens the scope of applicability of our model to real software systems.

- $\mu(\cdot) = \mu$ and $\rho(\cdot) = \rho$.
  In the simplest case, the service rate, as well as the failure rate, are constants. This implies that there is no performance degradation and the time to failure is exponentially distributed, which, because of its memoryless property, contradicts aging. Therefore, constants $\mu$ and $\rho$ do not capture the behavior of a software system which ages. This case will not be discussed further.

- $\mu(\cdot) = \mu(t)$ and $\rho(\cdot) = \rho(t)$
  In this case, the service rate and the failure rates are simply functions of time. Although arbitrary functions are allowed in the model, presumably, service rate will be a monotone nonincreasing function and the failure rate will be a monotone nondecreasing function of time in a software which ages. If

$$\rho(t) = \beta \alpha t^{\alpha-1},$$

where $\beta$ and $\alpha$ are constants with $\alpha > 1$, the time to failure has Weibull distribution with increasing failure rate, which is commonly used to model aging. To model software systems where only occasional failures are witnessed, with no performance degradation, the combination $\mu(\cdot) = \mu$ and $\rho(\cdot) = \rho(t)$ may be used. Further, to model software systems which undergo performance degradation, but are always available, the special case of $\rho(\cdot) = \rho = 0$ and $\mu(\cdot) = \mu(t)$ can be used.

- $\mu(\cdot) = \mu(N(t))$ and $\rho(\cdot) = \rho(N(t))$
  The service rate and the failure rate are functions of instantaneous load on the system, i.e., their value at time $t$ depends on the number of transactions in the queue at that time. This dependence is useful in capturing overload effects which especially influence the failure behavior. Of course, more realistic dependence on time, as well as instantaneous load, $\mu(\cdot) = \mu(t, N(t))$ and $\rho(\cdot) = \rho(t, N(t))$, is also allowed.

- $\mu(\cdot) = \mu(L(t))$ and $\rho(\cdot) = \rho(L(t))$
  A more complex, but also more powerful, dependence can be obtained by making $\rho(\cdot)$ and $\mu(\cdot)$ as functions of mean accumulated work done by the software system in a given time interval. Let $p_i(t), 0 \leq i \leq K$ be the probability that there are $i$ transactions in the queue at time $t$, given that the software is in state “A.” When the software is not aged, incoming transactions are promptly served and the total amount of time spent in actual processing in an interval $(0, t]$ is usually less than the interval $t$ itself. Since an idle software is not likely to age, service and failure rates are, more realistically, a function of the actual processing time rather than the total available time. Let $L(t)$ be defined as:

$$L(t) = \int_{\tau=0}^{t} \sum_i c_i p_i(\tau)d\tau,$$

where $c_i$ is a coefficient which expresses how big the state influences the degradation of the overall system. If $c_i = 0$ and $c_i = 1$ for $i > 0$, then $L(t)$ represents the average amount of time the software is busy processing transactions in the interval $(0, t]$. If $c_i = 1$ for $i \geq 0$, then $L(t) = t$ given that the software is available.

Last, our model allows for combination of the above dependencies. For example, the failure rate may be a function of not only the mean processing time in the interval $(0, t]$, but also of the instantaneous load at time $t$ to account for overload effects. In this case, $\rho(\cdot) = \rho(N(t), L(t))$.

In the following section, we derive the three measures for the two PM policies. Table 2 lists the notation used in the rest of the paper. (R.V. denotes random variable.)

### 4 Evaluation of Measures

Let the steady state availability of the software system be denoted by $A_{SS}$. Let $P_{los}$ denote the long run probability that an arriving transaction will be lost and let $T_{rep}$ denote the expected response time of a transaction, given that it is successfully served. The approach we follow in deriving the expressions for the three measures applies to both policies I and II. Only when a particular expression is different will it be noted explicitly. The solution method, in general, and the class of stochastic process used to model, in particular, provide an elegant, concise, and fast alternative to usually expensive discrete-event simulation approach.

As described in the previous section, the software can be in any one of three states at any time $t$. It can be up and available for service (state $A$), recovering from a failure (state $B$), or undergoing PM (state $C$) (see Fig. 1). Let $(Z(t), t$
Regeneration instants are embedded at times when the
also discarded. It can be shown that the process \(\{Z(t), t \geq 0\}\) is a Markov regenerative process (MRGP) [6]. The
stochastic process \(\{Z(t), t \geq 0\}\) is a Markov regenerative process (MRGP) [6]. The
expected time to recover from failure
\(\gamma_f\) Expected time to perform PM
\(U\) Sojourn time in state \(A\) (R.V.)
\(\lambda\) Transaction arrival rate
\(\rho()\) Failure rate
\(N(t)\) Number of transaction in the queue at time \(t\)
\(L(t)\) Mean processing time since the last renewal

\[ A_{SS} = \frac{\pi_A E[U]}{P_{AB} \gamma_f + P_{AC} \gamma_r + E[U]} \]  

The probability that a transaction is lost is defined as the ratio of expected number of transactions which are lost in an interval to the expected total number of transactions which arrive during that interval. The evolution of \([Z(t), N(t)], t > 0\) in state \(A\). Let \(E[U]\) denote its expectation. Expected sojourn times of the MRGP in states \(B\) and \(C\) are already defined to be given by \(\gamma_f\) and \(\gamma_r\). The steady state availability can then be obtained using standard formulae from MRGP theory [6] and is given as:

\[ L(t) \begin{cases} 0 & \text{if } t < \tau, \\ 1 & \text{if } t \geq \tau, \end{cases} \]  

The expected number of transactions lost is given by the summation of three quantities:

1) the expected number lost due to discarding because of failure or initiation of PM,
2) the expected number lost while recovery or PM is in progress, and
3) the expected number lost due to the buffer being full.

The last quantity is of special significance as, due to the degrading service rate, the probability of the buffer being full increases.

The probability of loss is then given by:

\[ P_{loss} = \frac{\pi_A E[N]}{\lambda(P_{AB} \gamma_f + P_{AC} \gamma_r + \int_0^\infty p_k(t) dt)} \]  

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where:

- \( E[N_i] \) is the expected number of transactions in the buffer when the system is exiting state \( A \);
- \( \lambda \gamma_i \) is the expected number of transactions arriving while the system is recovering;
- \( \lambda \gamma_i \) is the expected number of transactions arriving while the system is undergoing PM;
- \( \lambda \int_0^\infty p_k(t) dt \) is the expected number of transactions denied service because of the buffer being full while the system is in state \( A \);
- \( (\pi_B \gamma_i + \pi_C \gamma_i + \pi_A[E(U)]) \) is the average length of time between two consecutive visits to state \( A \).

Equation (4) is valid only for policy II. Under policy I the upper limit in the integral \( \int_0^\infty p_k(t) dt \) is \( \delta \) instead of \( \infty \). This is because the sojourn time in state \( A \) is limited by \( \delta \) under policy I.

We now derive an upper bound on the mean response time of a transaction given that it is successfully served, denoted by \( T_{res} \). The mean number of transactions, denoted by \( E \), which are accepted for service while the software is in state \( A \), is given by the mean total number of transactions which arrive while the software is in state \( A \) minus the mean number of transactions which are not accepted due to the buffer being full. That is,

\[
E = \lambda \left( E[U] - \int_0^\infty p_k(t) dt \right).
\]

Out of these transactions, on the average, \( E[N_i] \) are discarded later because of failure and initiation of PM. Therefore, the mean number of transactions which actually receive service, given that they were accepted, is given by \( E - E[N_i] \). The mean total amount of time the transactions spent in the system while the software is in state \( A \) is:

\[
W = \int_0^\infty \sum_i ip_i(t) dt.
\]

This time is composed of the mean time spent by the transactions which were served, as well as those which were discarded, denoted as \( W_s \) and \( W_{d} \) respectively; therefore, \( W = W_s + W_d \). The response time we are interested in is given by

\[
T_{res} = \frac{W_s}{E - E[N_i]},
\]

which is upper bounded by

\[
T_{res} < \frac{W}{E - E[N_i]}.
\]

Regardless of the PM policy, as can be observed from (3) and (4), we need to obtain expected sojourn times and the steady state probability of the software in each of the three states \( A, B, \) and \( C \), as well as the transient probability that there are \( i, i = 0, 1, \ldots, K \) transactions queued up for service. It is the last quantity which forbids a closed form analytical solution and necessitates a numerical approach.

The mean sojourn time in states \( B \) and \( C \) is already available as \( \gamma_i \) and \( \gamma_i \), respectively. The quantities still to be derived are related with the queuing behavior of the software in state \( A \), viz., \( p_{AB}, p_{AC}, E[U], \) and \( p_i(t), i = 0, 1, \ldots, K \). Their evaluation depends on the policy used.

### 4.1 Behavior of the System in State \( A \) Under Policy I

For \( Z(t) = A \), the subordinated process, i.e., the process until a regeneration occurs, is determined by the queuing behavior of the software processing transactions. The process is terminated by either a failure (which can happen at any time) or by initiating PM, which, under policy I, happens at time \( \delta \) if the software has not failed by that time. Fig. 3 shows the state diagram of the subordinated nonhomogeneous process under policy I. It is a birth-death process augmented with one absorbing state associated with each state of the birth-death process. Not included in the figure is the fact that, at \( t = \delta \), the subordinated process is terminated if it was not terminated before by a transition to an absorbing state \((0', \ldots, K')\).

By our notation, \( p_i(t) \) is the probability that there are \( i \) transactions queued for service, which is also the probability of being in state \( i \) of the subordinated process at time \( t \). Note that state \( i, i = 0, 1, \ldots, K \) is not to be confused with state \( i' \), \( i = 0, 1, \ldots, K \), which was defined just to be able to evaluate the quantities of interest. As such, all the states under the shaded area of the process can be lumped into a single absorbing state.

\[
p_i(t), i = 0, 1, \ldots, K \text{ and } p_j(t), i' = 0', 1', \ldots, K' \text{ can be obtained by solving the following system of forward differential-difference equations:}
\]

\[
\frac{dp_i(t)}{dt} = \mu_i p_{i-1}(t) + \lambda p_i(t) - (\mu_i + \lambda) p_i(t),
\]

\[
\frac{dp_{i-1}(t)}{dt} = \mu_i p_i(t) + \lambda p_{i-1}(t) - (\mu_i + \lambda + \rho_i) p_{i-1}(t),
\]

\[
1 \leq i < K
\]

\[
\frac{dp_{K-1}(t)}{dt} = \rho_K p_{K-1}(t) - (\mu_i + \rho_i) p_{K-1}(t)
\]

\[
\frac{dp_K(t)}{dt} = \rho_K p_K(t), \quad 0 \leq i \leq K.
\]

\( \mu_i \) and \( \rho_i \) can have any of the forms described in Section 3.1. If \( \mu_i = \mu(t) \) or \( \mu(N(t)) \) and \( \rho_i = \rho(t) \) or \( \rho(N(t)) \), no other changes, except for plugging in the proper values, is necessary. For \( \mu_i = \mu(L(t)) \) and \( \rho_i = \rho(L(t)) \), where \( L(t) \) is defined by

5. The measures evaluated in this paper require only the first moments of \( Y_t \) and \( Y_r \), and, hence, no assumptions on the nature of their distribution is made.
the set of ODEs is first augmented by the following differential equation:

\[
\frac{dL(t)}{dt} = \sum_i c_i p_i(t)
\]

and then solved.

The set of simultaneous differential-difference equations given by (6) do not, in general, have a closed-form analytical solution and must be evaluated numerically, along with the initial conditions \( p_i(0) = 1, p_i(0) = 0, 1 \leq i \leq K \), and \( p_i(0) = 0, 0' \leq i' \leq K' \). Once these probabilities are obtained, the rest of the quantities can be computed as follows:

One step transition probability \( P_{AB} \) is given by:

\[
P_{AB} = \sum_{i=0}^{K'} p_i(\delta)
\]

and

\[
P_{AC} = 1 - P_{AB}.
\]

Thereafter, according to (3), the steady state probability that the software is in states \( B \) and \( C \) can be obtained.

The expected sojourn time in state \( A \) is given by:

\[
E[N_A] = \sum_{i=0}^{K} \left( p_i(\delta) + p_i(\delta) \right)
\]

and the upper bound on \( T_{rep} \) as given in (3), (4), and (5), respectively, can now be easily calculated.

4.2 Behavior of the System in State \( A \) Under Policy II

If policy II is assumed, the evolution of the system in state \( A \) is somewhat more complex. In this case, we need to distinguish between \( t \leq \delta \) and \( t > \delta \), as policy II assumes that PM will be initiated if and only if the buffer is empty after time \( \delta \) has elapsed. For \( t \leq \delta \), exactly the same process of Fig. 3 determines the behavior of the software. For \( t > \delta \), the process which models the behavior is shown in Fig. 4. As can be observed, state 0 now belongs to the set of absorbing states, because PM will be initiated once the system becomes idle, thus terminating the subordinated process.

The set of forward differential-difference equations which are solved to determine all transient probabilities are given as follows:

\[
\frac{dp_i}{dt} = \mu(.) p_i(t) = \left( \lambda'(t) + \rho(.) \right) p_i(t)
\]

where \( \lambda'(t) = \lambda, \) if \( t \leq \delta \); otherwise, it is zero. Similarly, \( \rho'(t) = \rho, \) if \( t \leq \delta \); otherwise, zero. As before, \( \mu(.) \) and \( \rho(.) \) can be functions of \( t, N(t), \) or \( L(t) \), where, in the last case, the set of ODEs must be augmented with

\[
\frac{dL(t)}{dt} = \sum_i c_i p_i(t)
\]

and then solved. This set of differential-difference equations, along with the initial condition \( p_i(0) = 1, \) also requires numerical solution.

On step transition probability \( P_{AB} \) is computed by solving the system of ODEs at \( t = \infty \) and is given as:

\[
P_{AB} = \sum_{i=0}^{K'} p_i(\infty).
\]

Then

\[
P_{AC} = 1 - P_{AB} = p_0(\infty).
\]

The mean sojourn time in state \( A \) is now given by:

\[
E[N_A] = \sum_{i=0}^{K} \left( \sum_{i=0}^{K} p_i(t) \right) dt + \sum_{i=0}^{K} \left( \sum_{i=1}^{K} p_i(t) \right) dt
\]

\[
= \int_0^{\infty} p_0(t) dt + \int_0^{\infty} \sum_{i=0}^{K} p_i(t) dt.
\]

The mean number of transactions already in the queue the State \( A \) is exited is given by:

\[
E[N_A] = \sum_{i=0}^{K} \int_0^{\infty} p_i(\infty).
\]

Using (3), (4), and (5), the steady state availability, the probability of loss of an arriving transaction, and the upper bound on the response time of a transaction can now be calculated.
5 **NUMERICAL EXAMPLES**

In this section, we illustrate the usefulness of the models developed to evaluate $A_{ss}$, $P_{loss}$, and the upper bound on $T_{rev}$. The models are solved for multiple values of $\delta$ (PM interval in the case of policy I and PM wait in the case of policy II) and optimal values are determined. We also show how the optimum value of $\delta$ may be selected based on combined measures of the above three quantities.

Table 3 shows the parameter values that were kept fixed for all results. The values chosen are for illustration purposes only and do not necessarily represent any physical system.

The set of differential-difference equations given in (6) and (8) for policies I and II, respectively, were numerically solved using the LSODE routine in ANSI FORTRAN. LSODE (Livermore Solver for Differential Equations) is an ODE solver which uses Backward Differentiation Formula (BDF) methods for stiff systems of ODEs. It is publicly available as part of ODEPACK from netlib. A single run of the model takes less than a minute. A solution using commercially available packages like Mathematica or MATLAB is also possible, but is likely to be much slower. For large buffer sizes of the order of thousands, sparse matrix methods will need to be used in the ODE solution.

5.1 Experiment 1

In this experiment, $\gamma$ is varied to ascertain the effect of the measures and on optimal $\delta$. Service rate and failure rate are assumed to be functions of real time, i.e., $\mu(t) = \mu(t)$ and $\rho(t) = \rho(t)$, where $\rho(t)$ is defined to be

$$\rho(t) = \beta \alpha (t)^{-1},$$

which is the hazard function of Weibull distribution. $\alpha$ is fixed at 1.5 and $\beta$ is calculated from the MTTF and $\alpha$ as

$$\beta = \left[ \frac{\Gamma(1 + \frac{1}{\alpha})}{MTTF} \right]^{\alpha}.$$

(8)

The model is solved for both policies to show the effect of the cost of PM on the three measures, $\mu(t)$ is defined to be a monotone nonincreasing function of $t$, as shown in Fig. 5. This behavior of $\mu(t)$ has been given in [3] as an approximation to service degradation in telecommunications switching software.

$\mu_{\text{max}}$ is fixed at 15 hour$^{-1}$ and $\mu_{\text{min}}$ at 5 hour$^{-1}$ for all the experiments in the paper. For this particular experiment of varying $\gamma$, $\mu(t)$ (as shown in Fig. 5) is defined as:

$$\mu(t) = \begin{cases} \mu_{\text{max}} \left[ 1 - \frac{1}{\alpha} \right], & \text{if } t \leq a \\ \mu_{\text{min}}, & \text{if } t > a, \end{cases}$$

where

$$a = \left( \frac{\mu_{\text{max}} - \mu_{\text{min}}}{\mu_{\text{max}}} \right) \frac{MTTF}{\alpha}.$$
to policy I. This clearly shows that if the objective is to minimize long run probability of loss, which is the case in telecommunication switching software, policy II always fares better than policy I. It can also be observed, from Fig. 6a and 6b, that the value of $d$ which minimizes probability of loss is much lower than the one which maximizes availability. In fact, the probability of loss becomes very high at values of $d$ which maximize availability. Although the behavior is dependent on system parameter values, caution in proper selection of $d$ is indicated.

5.2 Experiment 2

In this experiment, $\gamma$ is fixed at 0.15. $\mu(\cdot) = \mu(t)$ and has exactly the same definition as in Experiment 1, $\rho(\cdot) = \rho(t)$, with previously defined Weibull hazard rate, except that the shape parameter $\alpha$ is now varied. Thus, for each value of $\alpha$, corresponding $\beta$ is calculated using (8), keeping MTTF fixed at 240 hours. In effect, we are interested in studying how the measures and the optimality vary as the failure density gets peakier with the same mean time to failure. $\alpha$ is assigned values of 1.0, 1.5, and 2.0, respectively.

Fig. 7a shows the steady state availability under the two policies plotted against $d$ for different values of $\alpha$. For $\alpha = 1.0$, the time to failure has an exponential distribution, which, because of its memoryless property, contradicts aging. As seen from the figure, it is better not to perform PM in this case if the objective is to maximize availability. For the other two values of $\alpha$, however, PM maximizes availability at certain $d$. For a specific policy, the peakier the failure density, i.e., higher the value of $\alpha$, the higher is the maximum steady state availability. Also, with higher values $\alpha$, this maxima occurs at lower values of $d$. Fig. 7b shows the long run probability of loss of a transaction plotted against $d$. In this case, PM proves to be beneficial for all three values of $\alpha$. Similar observations and arguments as those given in Experiment 1 also hold for this case.

5.3 Experiment 3

The purpose of this experiment is to illustrate the effect of assumptions on $\mu(\cdot)$ and $\rho(\cdot)$ on the three measures. Figs. 8a, 8b, and 8c show steady state availability, probability of loss, and the upper bound on the mean response time of transactions successfully served plotted against $d$ under policy I.

Each of the figures contains three curves. The solid curve represents a system, where $\mu(\cdot) = \mu(t)$ and $\rho(\cdot) = \rho(t)$. The dotted curve represents a second system, where $\mu(\cdot) = \mu(L(t))$ and $\rho(\cdot) = \rho(L(t))$. The parameters, $\mu_{\text{max}}, \mu_{\text{min}}, \mu_{\text{avg}}$, and $\alpha$ are kept the same. Similarly, $\alpha$ is kept at 1.5 for both the solid, as well as the dotted, curve. In other words, $\mu(\cdot)$ and $\rho(\cdot)$ in the solid curve are functions of real time, whereas, in the dotted curve, they are functions (with the same parameters) of the mean total processing time. The dashed curve represents a third system in
which no crash/hang failures occur, i.e., \( \rho(t) = 0 \), but service degradation is present with \( \mu(t) = \mu(0) \) with the same parameters as for \( \mu(t) \) of the earlier two systems. \( \gamma \) for all is kept fixed at 0.15. This experiment only illustrates the importance of making the right assumptions in capturing aging because, as seen from the figure, depending on the forms chosen for \( \mu(t) \) and \( \rho(t) \), the measures vary in a wide range.

Fig. 8 plots the upper bound on the real time performance. This was not shown for Experiments 1 and 2 because of its monotone nature. In the type of system under consideration, where queued transactions, as well as those arriving during recovery or PM, are lost, response time of successful transactions can be trivially minimized by always keeping the software unavailable (with very low success rate). This, however, will result in unacceptable values of the probability of loss and steady state availability. In many systems, for example, ATM switches, QOS requirements are specified using bounds on response time as well as probability of loss. This can be achieved via our model. For example, consider the solid curve in Fig. 8c. If QOS demands that the response time be less than 0.113 hours, \( \delta \) is restricted approximately in the interval \([0, 70]\). Now, if the QOS further demands that the probability of loss be minimized, the optimal \( \delta \) corresponds to the minimum \( P_{\text{loss}} \) within this interval only. The curve identified with the legend \((I, 1.5)\) in Fig. 7b plots \( P_{\text{loss}} \) against \( \delta \) with exactly the same parameter values and it is seen that global minima for \( P_{\text{loss}} \) occurs at \( \delta = 85 \), whereas the optimal \( \delta \) for the combined QOS is 70. Fig. 8b shows that the loss probability for no failure case is greater for the “no failure” case for higher values of \( \delta \). The reason is that the loss due to the buffer being full increases due to service degradation and a larger rejuvenation interval. In the other two cases, although a failure results in the loss of queued transactions, it also restores the service rate to the peak value, thereby reducing the overall probability of the buffer being full.

### 6 Conclusion

In this paper, we motivated the need for pursuing preventive maintenance in operational software systems on a scientific analytical basis rather than the current ad hoc practice. We presented a model for a transactions based software system which employs preventive maintenance to maximize availability, minimize probability of loss, minimize response time, or optimize a combined measure. We evaluated the three measures for two different preventive maintenance policies and showed via numerical examples that a policy which takes into account instantaneous load on the system results in lower optimum probability of loss. The effect of aging is captured as crash/hang failures, as well as performance degradation. Systems which experience only one of the two can be modeled as special cases. The main strength of our model, however, is its capability of capturing the dependence of crash/hang failures and performance degradation on time, instantaneous load, mean accumulated load, or a combination thereof. This, in our opinion, provides great flexibility in modeling real situations and widens the scope of applicability of our model. The main limitation, on the other hand, is that it is applicable to only those software systems in which incoming transactions are lost when either it fails or when PM in initiated. In many database systems which support recovery, transactions are logged when they arrive. Even when failure occurs, the log is not lost, thus violating our assumption.

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This work was done while Sachin Garg was a graduate student in the Department of Electrical and Computer Engineering at Duke University, Durham, North Carolina.

### References


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